

Writing Biconditional Statements // Truth Tables

Lesson Objective | NBAT write biconditional statements; write definitions as biconditional statements; fill out truth tables to find logically equivalent statements.

Using Definitions

You can write a definition as a conditional statement in "if-then" form or as its converse. Both the conditional statement AND converse are true for definitions. For example, consider the following definition of *perpendicular lines*:

P $\rightarrow 90^\circ$ is not included in def.

If two lines intersect to form a right angle, then they are perpendicular lines.

Now, write the converse of the definition:

If 2 lines are perp., then they intersect to form a right angle.

Example: Use the diagram. Decide whether each statement is true. Explain your answer using the definitions you have learned from Chapter 1 and above.

1. $\angle JMF$ and $\angle FMG$ are supplementary.

True - straight $\angle = 180^\circ$, angle addition postulate

2. Point M is the midpoint of $\overline{FH}$.

False - if $\overline{FM} \cong \overline{MH}$, then M not midpoint.

3. $\angle JMF$ and $\angle HMG$ are vertical angles.

True - if 2 pairs of opp. rays, then vertical angles.

4. $\overline{FH} \perp \overline{JG}$

False - (b) if no right angle, then no $\perp$ lines.

Biconditional Statements: When a conditional statement and its converse are both TRUE, you can write them as a single biconditional statement.

$\rightarrow$ A biconditional statement is a statement that contains the phrase "if and only if"

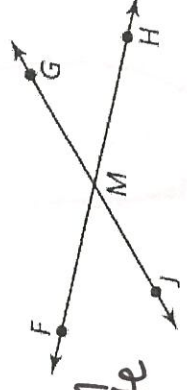
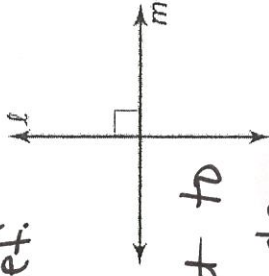
Words: p if and only if q Symbols: $p \leftrightarrow q$

**We saw above that definitions have true conditional and converse statements. So, ANY definition can be written as biconditional statements!

$\star p \leftrightarrow q$ come from orig. conditional!

Example: Rewrite the definition of perpendicular lines above as a single biconditional statement. (not converse)

2 lines intersect to form a right angle if and only if they are perpendicular lines.



6. Rewrite the definition of congruent segments (shown below) as a single biconditional statement:
 "If two line segments have the same length, then they are congruent segments."

2 line segments have the same length if and only if they are congruent segments.

7. Write the conditional statement and converse within the biconditional.

"An angle is a right angle if and only if its measure is 90° ."

$P \rightarrow Q$ Conditional: If an angle is a right angle, then its measure is 90° .

$Q \rightarrow P$ Converse: If an angle's measure is 90° , then it is a right angle.

Truth Tables

Truth Value: The truth value of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a truth table.

Let's look at conditional statements bring TRUE or FALSE in the context of keeping a promise.

Conditional			
P	q	$P \rightarrow q$	
T	T	T	
T	F	F	
F	T	T	
F	F	T	

MY PROMISE: If there is an elephant in the library, then I will give you \$100.

- Suppose there really IS an elephant in the library, and I give you \$100.
- Suppose there really IS an elephant in the library, and I don't give you any money.
- Suppose there is not an elephant in the library, but I give you \$100 anyway. (SWEET!)
- Suppose there is not an elephant in the library, and I don't give you any money.

Now, let's fill out the following truth tables for the converse, inverse, and contrapositive of a conditional statement:

Converse			
P	q	$q \rightarrow p$	
T	T	T	
T	F	T	
F	T	F	
F	F	T	

Inverse				
P	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive				
P	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Think About It... Two statements are logically equivalent when they have the same truth table.

So, which statements define the term above? (HINT: look at their truth table outcomes!)

converse and inverse
 conditional and contrapositive